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# Adaptive Testing for Connected and Automated Vehicles with Sparse Control Variates in Overtaking Scenarios

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# 目录

CONTENTS



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# 1 Introduction

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# 1 Introduction

- Many accidents of connected and automated vehicles (CAVs)

Tesla

Uber


Waymo

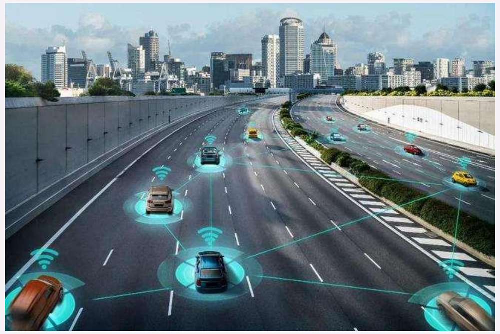
NIO

- Testing and evaluation

Testing
Evaluation

critical step →

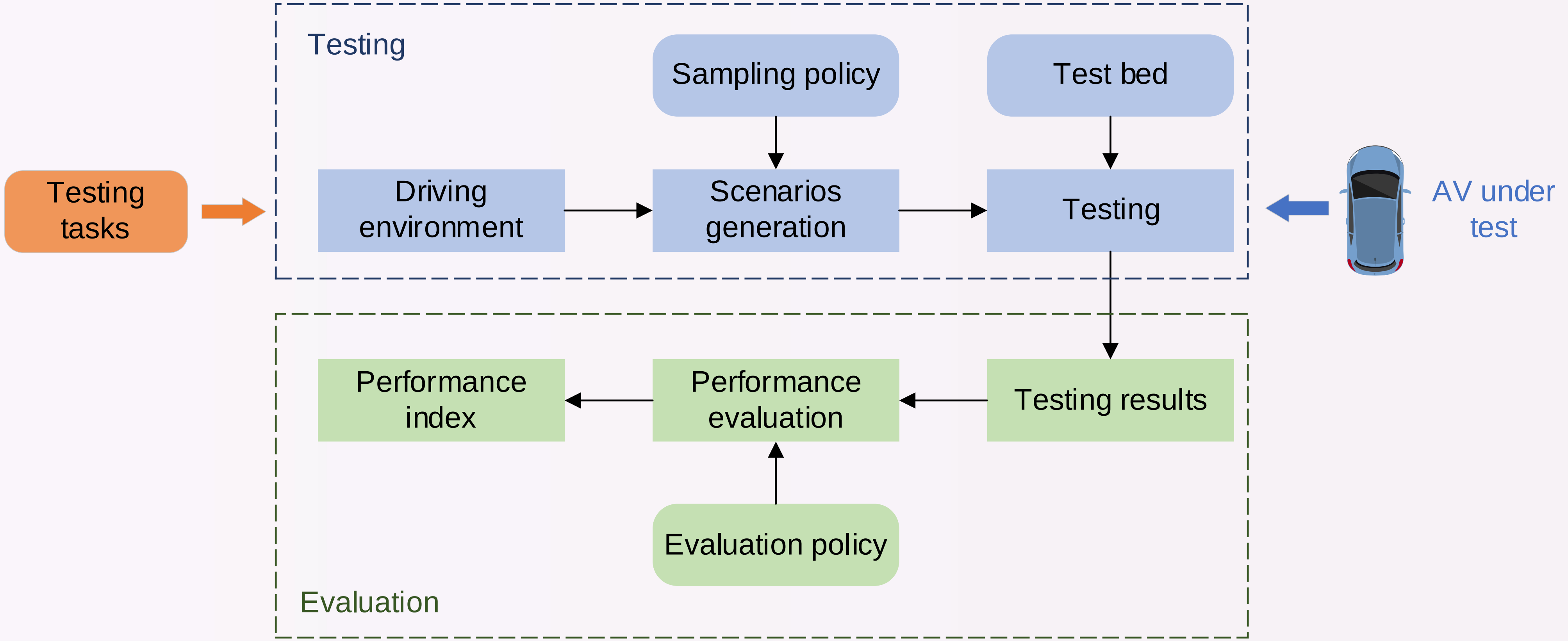
Development
Deployment



Source: Internet.

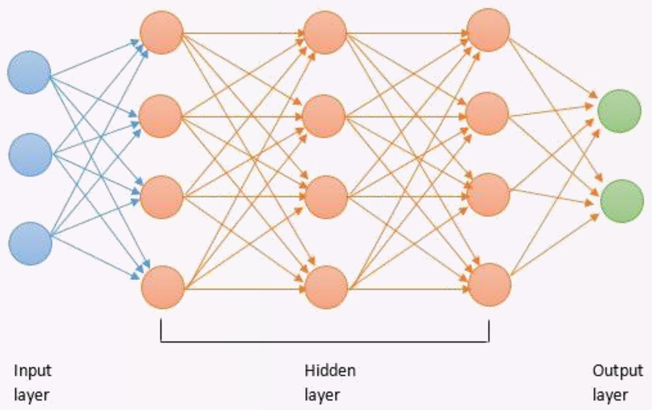
# 1 Introduction

- Testing and evaluation procedures



# 1 Introduction

- Main properties of CAVs




black-box

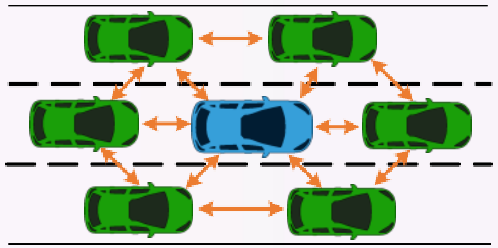
various types

- How to adaptively test and evaluate CAVs?

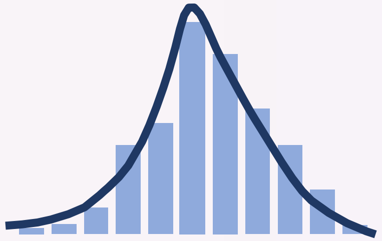
Source: Internet.

# 1 Introduction

- Adaptively generate testing scenarios
  - Existing several methods[Mullins, 2018; Koren, 2018; Sun, 2021; Feng, 2022]
- Adaptively evaluate testing results
  - Complementary to adaptively generating testing scenarios
- Challenge: high-dimensional scenarios



Interactivity



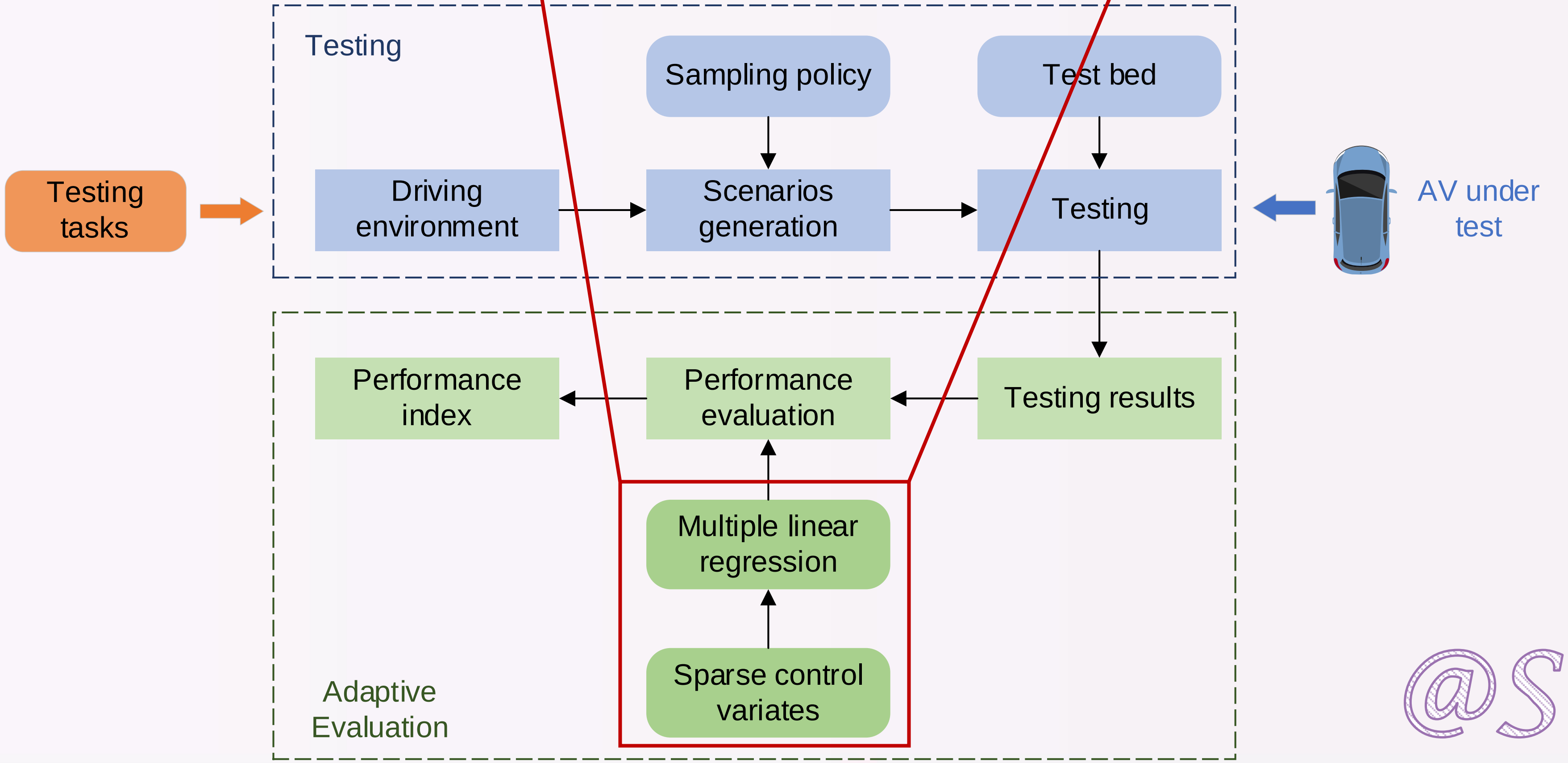
Randomness

Spatiotemporal complexity*

Source: *https://www.zf.com/mobile/en/stories_13632.html.

# 1 Introduction

- Adaptive testing with sparse control variates (ATSCV or @SCV)

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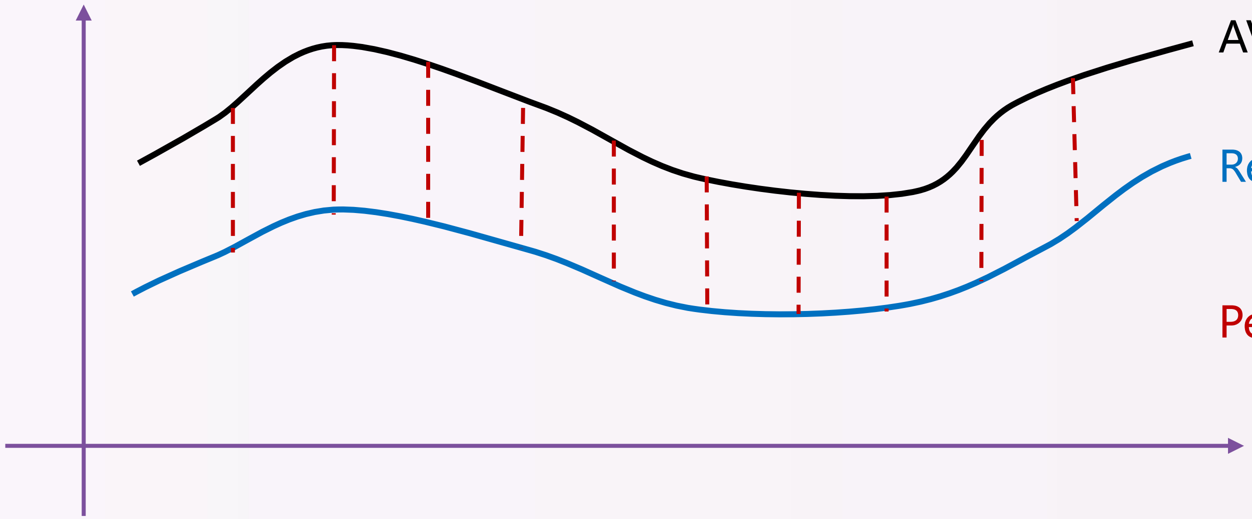
# 1 Introduction

- Intuition of ATSCV




- Case study: overtaking scenarios
- Accelerated rate: ≈ 30 times

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# 2 Problem Formulation

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# 2.1 Overtaking Scenarios

- Definitions
  - state: $s \triangleq \left(v_{\mathrm{BV}}, R_1, \dot{R}_1, R_2, \dot{R}_2\right) \in \mathcal{S}$
  - action: $a \triangleq a_{\mathrm{BV}} \in \mathcal{A}^+$
  - overtaking scenario: $x = (s_0, a_0, \ldots, s_m, a_m) \in \mathcal{X}$

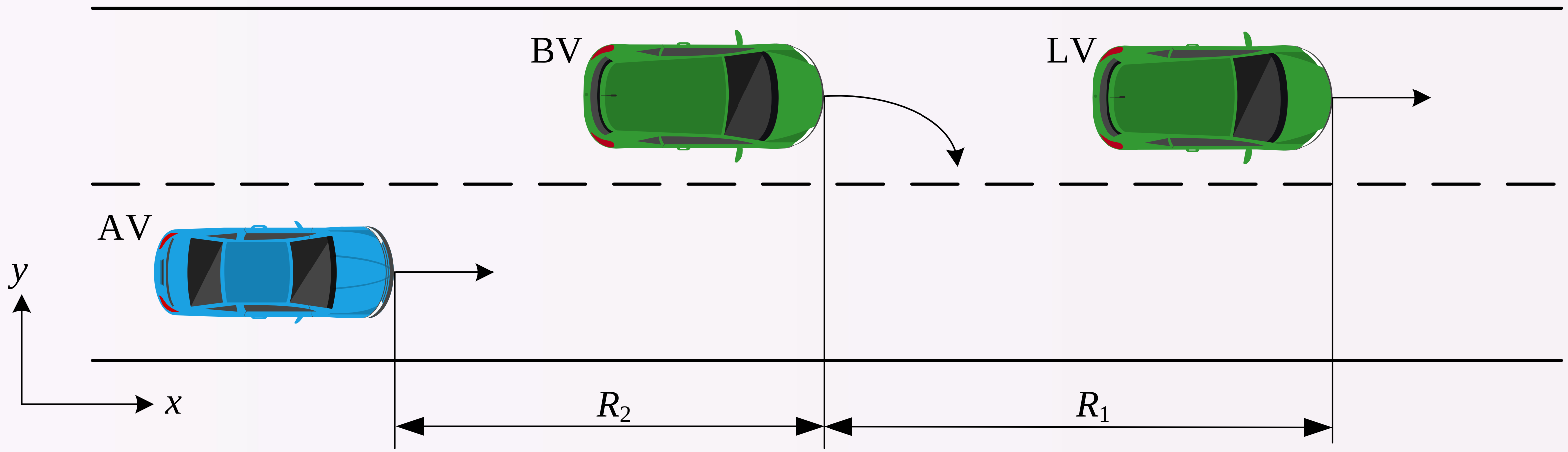
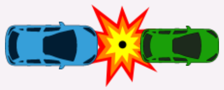



Fig. 1. Illustration of the overtaking scenarios.

# 2.2 Testing Scenario Library Generation

- Accident event: $A = \{x \in \mathcal{X} \mid R_{1,m} \leq d_{\text{accid}}\},\ d_{\text{accid}} = 1\ \text{m}$ 
- Accident rate:

$$\mu = \mathbb{E}_p[\mathbb{I}_A(X)] = \sum_{x \in \mathcal{X}} \mathbb{P}(A|x)\, p(x)$$

- Naturalistic distribution:

$$p(x) = p(s_0) \prod_{k=0}^{m} p(a_k|s_k)$$

- Naturalistic driving environment (NDE): Monte-Carlo simulation

$$\hat{\mu}_n = \frac{1}{n} \sum_{i=1}^{n} \mathbb{P}(A|X_i), \quad X_i \sim p$$

$$\text{Estimation} = \frac{\text{Number of accidents}}{\text{Number of tests}}$$

# 2.2 Testing Scenario Library Generation

- Long tail dilemma → Importance sampling (IS) [Zhao, 2016; Feng, 2021a,b]

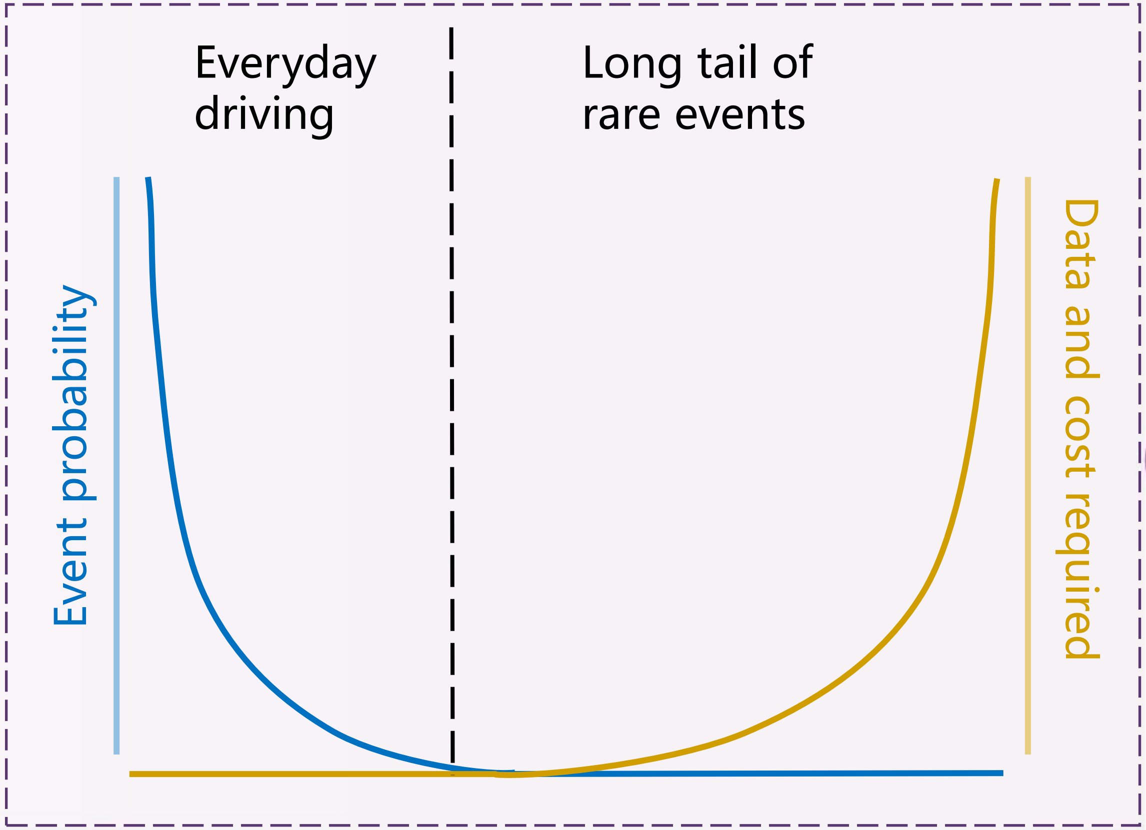
$$\mu = \sum_{x \in \mathcal{X}} \mathbb{P}(A|x)\, p(x)$$

$$= \sum_{x \in \mathcal{X}} \frac{\mathbb{P}(A|x)\, p(x)}{q(x)} q(x)$$

$$= \mathbb{E}_q \left[ \frac{\mathbb{P}(A|X)\, p(X)}{q(X)} \right]$$

$$\Downarrow$$

$$\hat{\mu}_q = \frac{1}{n} \sum_{i=1}^{n} \frac{\mathbb{P}(A|X_i)\, p(X_i)}{q(X_i)}, \quad X_i \sim q$$

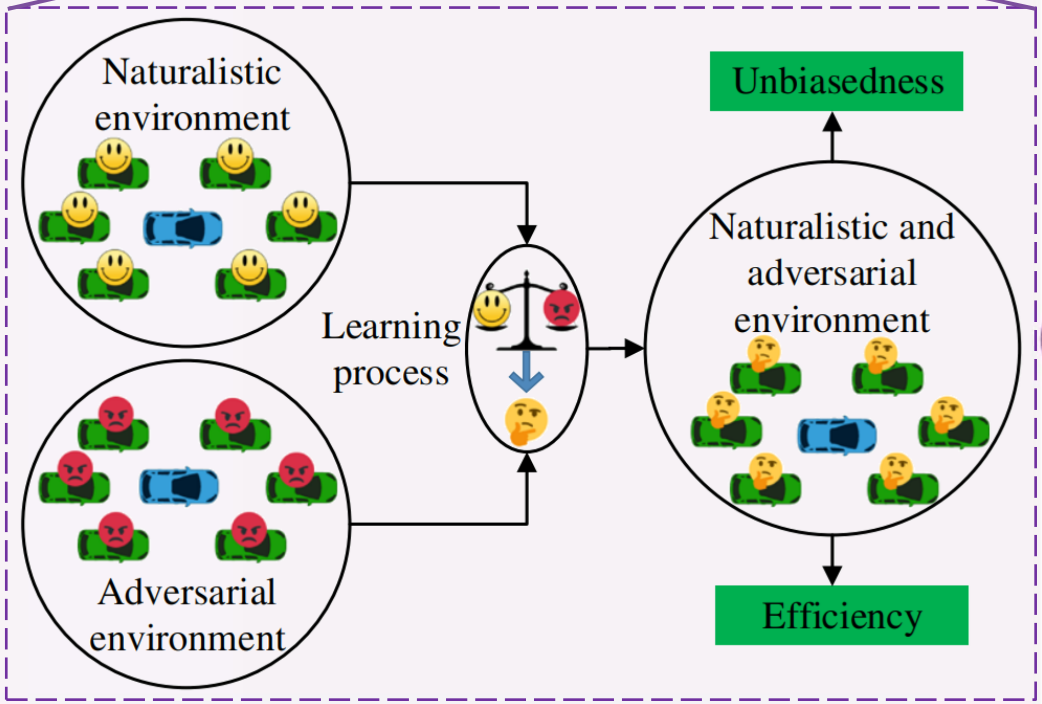



Source: https://blog.crossminds.ai/post/2019-ai-commercialization-conference-trends-challenges-featured-talks-autonomous-driving-nlp-robot-transportation.

# 2.3 Naturalistic and Adversarial Driving Environment Generation

- Curse of dimensionality (CoD)
  - Naturalistic and adversarial driving environment (NADE) [Feng, 2021c]
- NADE estimation[Feng, 2021c] :

$$\tilde{\mu}_q = \frac{1}{n}\sum_{i=1}^{n}\frac{\mathbb{P}(A|X_i)\,p(X_{c,i})}{q(X_{c,i})}, \quad X_i \sim q$$




Source: [Feng, 2021c].

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# 3 Adaptive Testing with Sparse Control Variates

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# 3.1 Control Variates

- Control variates (CVs): $h(x) = (h_1(x), \ldots, h_J(x))^\top, \ \sum_{x \in \mathcal{X}} h(x) = \theta$
- Mixture importance sampling: multiple importance functions

$$q_\alpha = \sum_{j=1}^{J} \alpha_j q_j, \ \alpha_j \geqslant 0, \ \sum_{j=1}^{J} \alpha_j = 1$$

- Mixture IS with CVs: component densities

$$\hat{\mu}_{q_\alpha,\beta} = \frac{1}{n} \sum_{i=1}^{n} \frac{\mathbb{P}(A|X_i)p(X_i) - \sum_{j=1}^{J} \beta_j q_j(X_i)}{\sum_{j=1}^{J} \alpha_j q_j(X_i)} + \sum_{j=1}^{J} \beta_j, \ X_i \sim q_\alpha$$

  - where $\beta_j$ are control parameters

# 3.1 Control Variates

- Minimize variance: $\min_{\beta} \mathrm{Var}_{q_\alpha}(\hat{\mu}_{q_\alpha,\beta}) \Rightarrow \beta^*$
- Property of variance[owen, 2000]:

$$\mathrm{Var}_{q_\alpha}\left(\hat{\mu}_{q_\alpha,\beta^*}\right) \leqslant \min_{1\leqslant j\leqslant J} \frac{\sigma_{q_j}^2}{n\alpha_j}$$

  - where

$$\sigma_{q_j}^2 = \mathrm{Var}_{q_j}\left(\frac{\mathbb{P}(A|X)p(X)}{q_j(X)}\right),\ j=1,\ldots,J$$

- Find the optimal control parameters:
  - multiple linear regression (MLR) of $Y_i$ on $Z_{ij}$

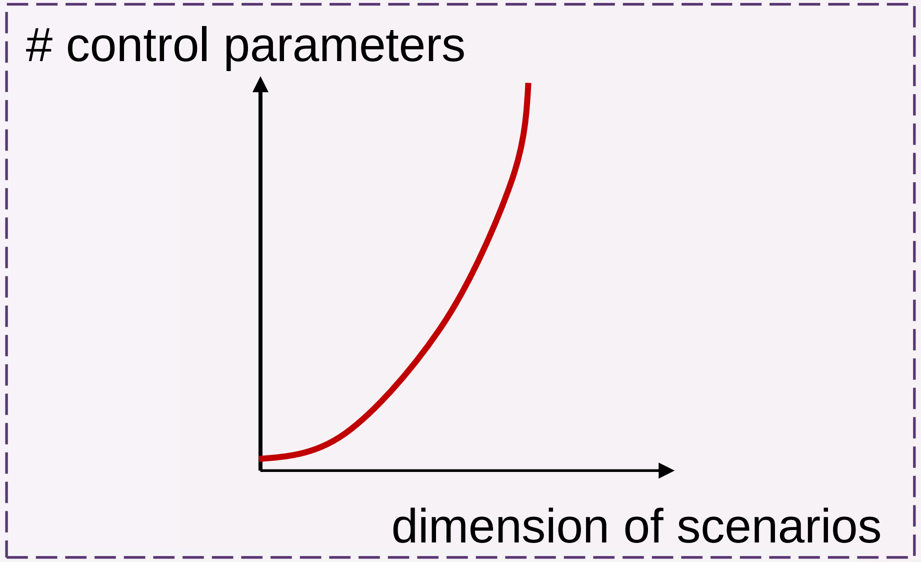
$$\hat{\mu}_{q_\alpha,\beta} = \frac{1}{n}\sum_{i=1}^{n}\left[\boxed{\frac{\mathbb{P}(A|X_i)p(X_i)}{q_\alpha(X_i)}} - \sum_{j=1}^{J}\beta_j\boxed{\left(\frac{q_j(X_i)}{q_\alpha(X_i)}-1\right)}\right],\ X_i\sim q_\alpha$$

# 3.2 CoD of Control Variates

- Mixture importance function:

$$q_\alpha(x) = q_\alpha(s_0)\prod_{k=0}^{m} q_\alpha(a_k|s_k),\ q_\alpha(s) = \sum_{j=1}^{J}\alpha_j q_j(s),\ q_\alpha(a|s) = \sum_{j=1}^{J}\alpha_j q_j(a|s)$$

- Control variates: $q_{j_0,\dots,j_{m+1}}(x) = q_{j_0}(s_0)\,q_{j_1}(a_0|s_0)\cdots q_{j_{m+1}}(a_m|s_m),\ \#\mathrm{CVs} = J^{m+2}$
- Curse of dimensionality:
    - solving optimal control parameters via MLR



# 3.3 Sparse Control Variates

- Sparse control variates (SCVs)
  - constructed by importance functions of critical variables
  - critical variables are sparse in NADE → sparse control variates
- Stratified scenario sets: $\mathcal{X}_l = \{x \in \mathcal{X} : |x_c| = l\},\ l = 0, 1, \ldots, L$
- Mixture IS: $X_i \sim q_\alpha$

$$\mu_l \triangleq \mathbb{E}_p[\mathbb{I}_A(X)\mathbb{I}_{\mathcal{X}_l}(X)] \quad \Rightarrow \quad \mu = \sum_{l=0}^{L} \mu_l$$

$$\left.\begin{aligned} \tilde{\mu}_{l,q_\alpha} &= \frac{1}{n}\sum_{i=1}^{n} \frac{\mathbb{P}(A|X_i)\mathbb{I}_{\mathcal{X}_l}(X_i)p(X_{c,i})}{q_\alpha(X_{c,i})} \\ \tilde{\mu}_{q_\alpha} &= \frac{1}{n}\sum_{i=1}^{n} \frac{\mathbb{P}(A|X_i)p(X_{c,i})}{q_\alpha(X_{c,i})} \end{aligned}\right\} \quad \Rightarrow \quad \tilde{\mu}_{q_\alpha} = \sum_{l=0}^{L} \tilde{\mu}_{l,q_\alpha}$$

# 3.3 Sparse Control Variates

- Sparse control variates (SCVs)
  - constructed by importance functions of critical variables

$$q_{j_1,\dots,j_l}(x_c) = q_{j_1}(x_{c_1}) \cdots q_{j_l}(x_{c_l})$$

- Denote

$$h_l(x_c) \triangleq \sum_{j_1,\dots,j_l} \beta_{j_1,\dots,j_l} q_{j_1,\dots,j_l}(x_c),\ \theta_l \triangleq \sum_{x \in \mathcal{X}_l} h_l(x_c)$$

- Adaptive testing with sparse control variates (ATSCV)

$$\tilde{\mu}_{l,q_\alpha,\beta_l} = \frac{1}{n} \sum_{i=1}^{n} \frac{\mathbb{P}(A|X_i)p(X_{c,i}) - h_l(X_{c,i})}{q_\alpha(X_{c,i})} \mathbb{I}_{\mathcal{X}_l}(X_i) + \theta_l \quad \Rightarrow \quad \boxed{\tilde{\mu}_{q_\alpha,\beta} = \sum_{l=0}^{L} \tilde{\mu}_{l,q_\alpha,\beta_l}}$$

# 4 Simulation Analysis

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# 4.1 Generation of NDE

- Settings
  - LV: constant speed
  - BV: intelligent driver model (IDM) for car-following and stochastic minimizing overall braking induced by lane changes (MOBIL) for cut-in
  - AV: constant speed for free-flow, IDM for car-following
  - Model parameters: from [Feng, 2021c]
  - Simulation frequency: 10 Hz (i.e., 0.1 s)
  - Initial states:

$$s_0 = [8, R_1, -5, 5, -5],\ R_1 \sim \mathcal{U}(30, 32)$$

# 4.2 Generation of NADE

- Maneuver criticality: $V(a|s) = \boxed{\mathbb{P}(A|s,a)}\ \boxed{p(a|s)}$

$$\mathbb{P}(S_j|s,a) = \sum_{a \in \mathcal{A}^+} \mathbb{P}(S_j|s',a)\, p(a|s')$$

Maneuver challenge

$$p(a|s) = \begin{cases} p_R, & \text{if } a = \Rsh \\ 1 - p_R, & \text{if } a = \mathrm{IDM}(s) \in \mathcal{A} \\ 0, & \text{otherwise} \end{cases}$$

Exposure frequency

- Importance function:

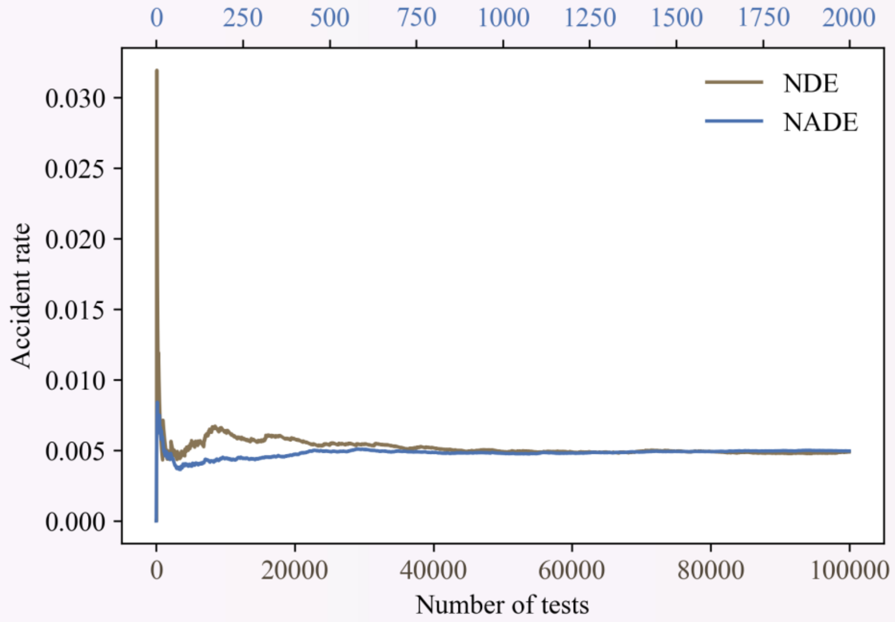
$$q_j(a|s) = \begin{cases} \epsilon p(a|s) + (1-\epsilon)\dfrac{V_j(a|s)}{C_j(s)}, & \text{if } C_j(s) > 0 \\ p(a|s), & \text{otherwise} \end{cases}$$

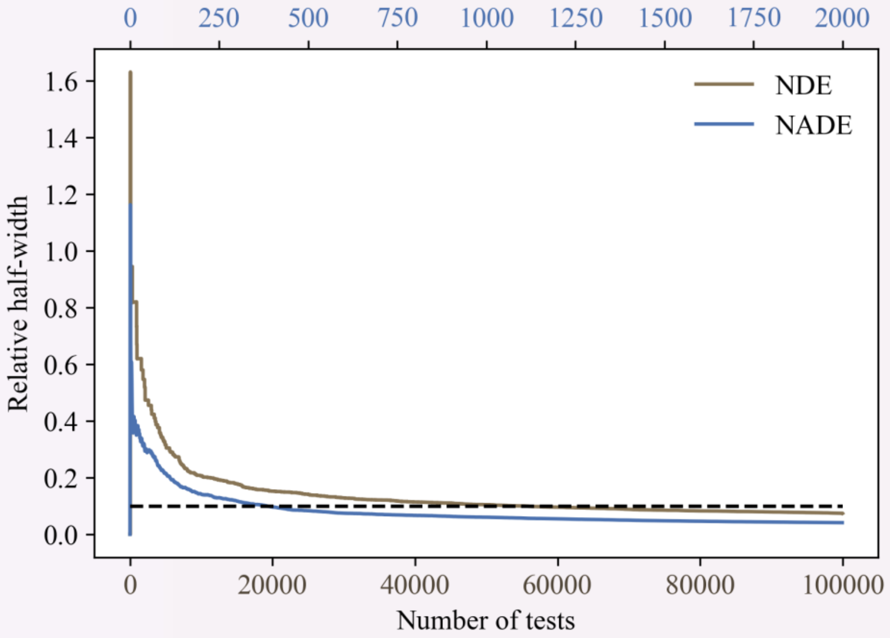
- where

$$C_j(s) = \sum_{a \in \mathcal{A}^+} V_j(a|s)$$

# 4.3 Evaluation Results

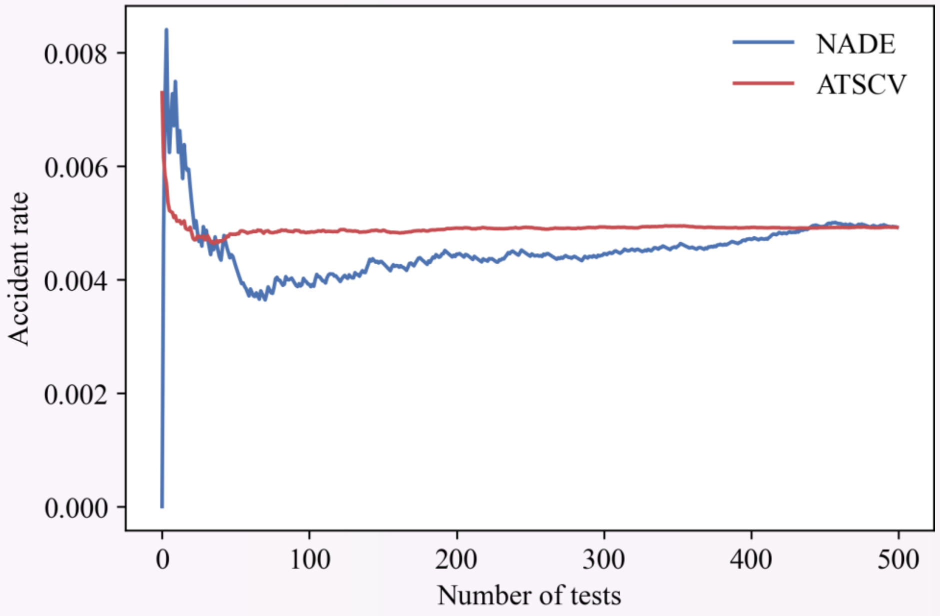
- NDE vs. NADE
- Accelerated rate ≈ 143 times

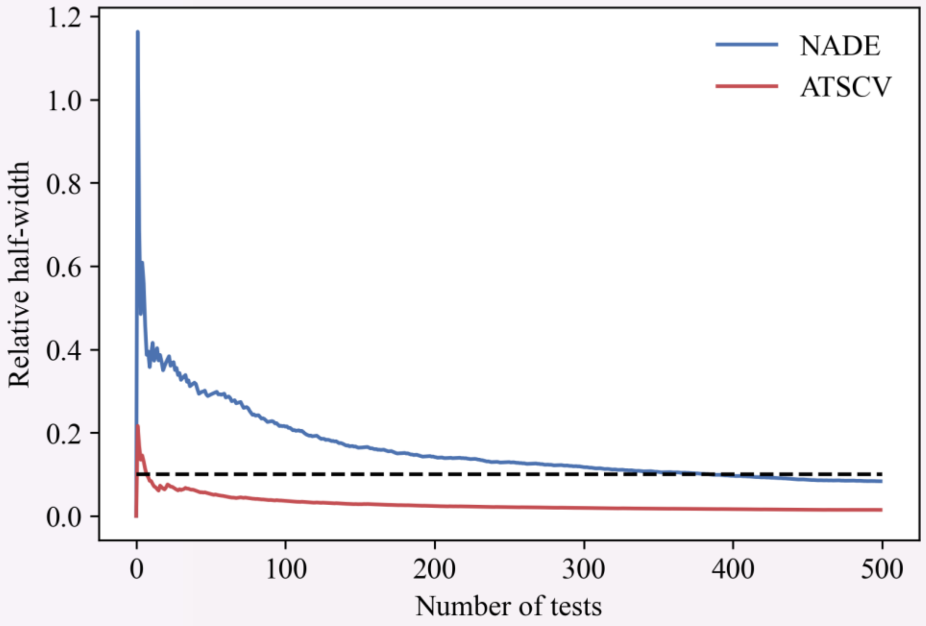


# 4.3 Evaluation Results

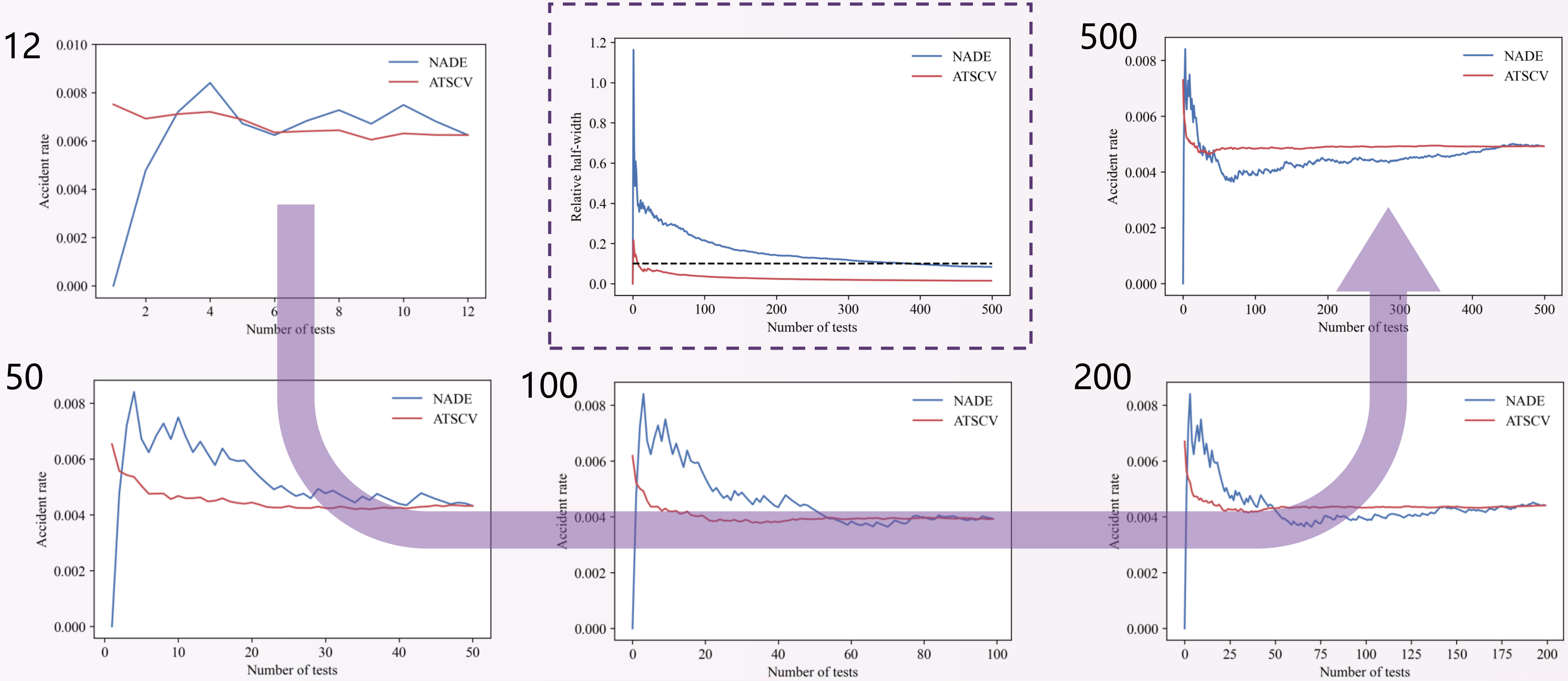
- NADE vs. ATSCV
- Accelerated rate ≈ 30 times



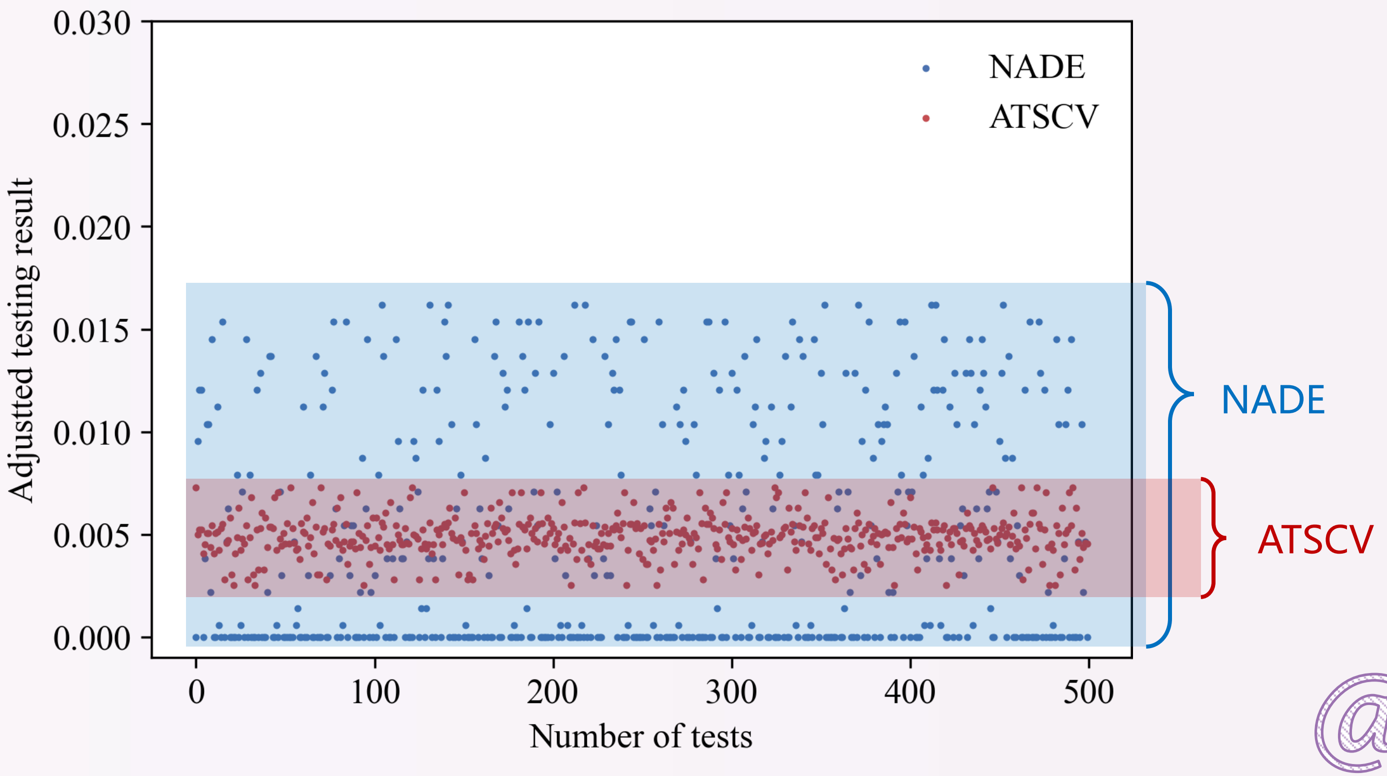
# 4.3 Evaluation Results

- NADE vs. ATSCV



# 4.3 Evaluation Results

- Adjusted testing results: NADE vs. ATSCV

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# 5 Conclusion

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# 5 Conclusion

- Problems: adaptively testing in high-dimensional scenarios
- Methods: adaptive testing with sparse control variates (ATSCV)
  - SCVs: importance functions of only critical variables
- Results: accelerated rate ≈ 30 times
- Future study:
  - theoretical analysis with rigorous proofs
  - realistic cases with large-scale naturalistic driving data

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# Thanks for your attention!

# Q&A

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